Differential Equation Question Bank

Definition:

An equation that contain one or more derivatives or differentials is called the differential equation.

- **Order:** The order of highest derivatives present in the equation determines the order of differential equation.
- **Degree:** The maximum power of highest derivatives determine the degree of the differential equation.

Note: First equation has been cleared from fractional and radical signs in the dependent variables and its derivatives.

Ex: $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^5 + y = e^x \quad \rightarrow \textit{order 3 and degree 2}$

$\left(\frac{dy}{dx}\right)^2 + 6\left(\frac{dy}{dx}\right) + 8y = \sin x \quad \rightarrow \textit{Order 1 and degree 2}$

Exercise No.01:

Que: find the order and degree of differential equation.

1) $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 - 6y = 0$
2) $\frac{d^2y}{dx^2} = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^3$
3) $\sqrt{\frac{d^3y}{dx^3}} + \frac{dy}{dx} = y$
4) $\sqrt{\frac{dy}{dx}} = \sqrt[3]{\frac{d^2y}{dx^2}}$
5) $\sqrt[3]{\frac{dy}{dx}} + y = \sqrt[4]{\frac{d^2y}{dx^2}}$
6) $\frac{d^2y}{dx^2} + \sqrt{1 + \frac{dy}{dx}} = 0$
7) $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 5\frac{d^2y}{dx^2}$
8) $\left(\frac{d^3y}{dx^3}\right)^3 + 2\left(\frac{d^2y}{dx^2}\right)^4 + 5\frac{dy}{dx} + 6y = 4$
9) $\frac{d^2y}{dx^2} = \left(y + \frac{dy}{dx}\right)^{\frac{3}{2}}$
10) $\frac{d^2y}{dx^2} = \sqrt{y - \frac{dy}{dx}}$
11) $\frac{d^2y}{dx^2} = \sqrt[3]{1 + \frac{dy}{dx}}$
12) $\frac{d^2y}{dx^2} = \sqrt[4]{y + \left(\frac{dy}{dx}\right)^2}$
13) $\sqrt[3]{\frac{d^2y}{dx^2} + 4x} = \sqrt{\frac{dy}{dx} - 1}$
14) $2\frac{d^2y}{dx^2} + \sqrt[3]{1 - \left(\frac{dy}{dx}\right)^2} - y = 0$
15) $\frac{d^3y}{dx^3} = \left[k + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$
16) $y = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$
17) $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{5}{3}} = 2\frac{d^2y}{dx^2}$
18) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \text{my}$

➢ Formation of differential equation

Exercise No.02:

1) Form the differential equation if, $y = ax^2 + b$.
2) Form the differential equation if, $y = 4(x - A)^2$. Where A is arbitrary constant.
3) Find the differential equation from the relation $y = Ae^{mx}$
4) Find the differential equation from the relation $y^2 = 4ax$
5) Form the differential equation if $y = \cos(x + a)$.
6) Form the differential equation from the equation $y = Ae^{3x} + Be^{-3x}$ by eliminating the arbitrary constants.
7) Form the differential equation if $x^2 + cy^2 = 4$.
8) Form the differential equation from the equation $y = Ae^{2x} + Be^{-2x}$ by eliminating the arbitrary constants.
9) Form the differential equation of $y = a\cos 4x + b\ sin\ 4x$
10) Form the differential equation of $y = A\cos 3x + B\ sin\ 3x$
11) Form the differential equation of $y = A\sin x + B\cos x$
12) Form the differential equation whose general solution is $y = A\cos(\log x) + B\sin(\log x)$, A and B are arbitrary constant

➢ Methods to solve differential equation

There are five methods such as follows

1) Variable Separable Form
2) Linear Differential Equation

Method no. 1. Variable Separable Form

By simple adjustment if it is possible to write all the term containing x along with dx and the term containing y along with dy, then the equation is said to be in variable separable form.

If $\quad f(x)dx = g(y)dy$

Then the direct integration of such an equation gives the general solution of the equation.

$i.e. \int f(x)dx = \int g(y)dy$ is a general solution.

Differential Equation Question Bank

Exercise No.03:

Q.1. Solve $e^{y}\frac{dy}{dx}=x^{2}$

Q.2. Solve $x\,dy-y\,dx=0$

Q.3. Solve $x^{2}dx=y^{2}\,dy$

Q.4. Solve $\sin x\cos y\,dy+\sin y\cos x\,dx=0$

Q.5. Solve $sec^{2}x\tan y\,dx+sec^{2}y\tan x\,dy=0$

Q.6. Solve $\frac{dy}{dx}=e^{x-y}+xe^{-y}$

Q.7. Solve $\frac{dy}{dx}=e^{3x-2y}+x^{2}e^{-2y}$

Q.8. Solve $\sqrt{1-y^{2}}dx-\sqrt{1-x^{2}}dy=0$

Q.9. Solve $(1+x^{2})dy=\sqrt{y}dx$

Q.10. Solve $3e^{x}\tan y\,dx+(1-e^{x})sec^{2}y\,dy=0$

Q.11. Solve $\frac{dy}{dx}=e^{(x-y)}x^{2}$

Q.12. Solve $\frac{dy}{dx}=\frac{1+x^{2}}{y}$

Q.13. Solve $(1+x^{2})dy-(1+y^{2})dx=0$

Q.14. Solve $\frac{dy}{dx}=e^{2x+y}+x^{2}e^{y}$

Q.15. Solve $\frac{dy}{dx}=e^{2x-3y}+4x^{2}e^{-3y}$

Q.16. Solve $x(1+y^{2})dx+y(1+x^{2})dy=0$

Q.17. Find the particular solution of D.E. $\frac{dy}{dx}=6-3x$. Given at $x=0, y=0$.

Q.18. Find the particular solu. of D.E. $y\sqrt{1-x^{2}}dy+x\sqrt{1-y^{2}}dx=0\ when\ x=\frac{3}{4}, y=\frac{4}{5}$

Differential Equation Question Bank

Method no. 2. Linear Differential Equation

- If equation in the form of

  $\frac{dy}{dx} + Py = Q$, where P and Q are constant or function of x only.

  Then its solution is

  $$y\, e^{\int P\, dx} = \int \left(Q\, e^{\int P\, dx}\right) dx + c$$

  $e^{\int P\, dx}$ is called as integrating factor.

- If equation is in the form of

  $\frac{dx}{dy} + Px = Q$, where P and Q are constant or function of y only

  Then its solution is

  $$x\, e^{\int P\, dy} = \int \left(Q\, e^{\int P\, dy}\right) dy + c$$

  $e^{\int P\, dy}$ is called as integrating factor.

Exercise No.04:

Q.1. Solve $x\frac{dy}{dx} + y = x^2$

Q.2. Solve $\frac{dy}{dx} + y\cot x = coces\, x$

Q.3. $x\frac{dy}{dx} - y = x^2 cos^2 x$

Q.4. Solve $(x^2+1)\frac{dy}{dx} + 2xy = \frac{1}{x^2+1}$

Q.5. Solve $(x^2+1)\frac{dy}{dx} + 2xy = 2x$

Q.6. Solve $\cos x\frac{dy}{dx} + 2y\sin x = \sin 2x$

Q.7. Solve $cos^2 x\frac{dy}{dx} + y = \tan x$

Q.8. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(1+x)^2$

Q.9. Solve $(x+1)\frac{dy}{dx} - y = e^{x}(1+x)^2$

Q.10. Solve $\frac{dy}{dx} + y\tan x = cos^2 x$

Q.11. Solve $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$

Q.12. Solve $\frac{dy}{dx} - y = 3e^{-2t}$ if $y(0) = -1$