Application of Derivatives Question Bank
2nd Semester Diploma Engineering for All Branch

Slope of Tangent and Normal

- ✓ Slope of tangent to curve at a point (x,y) is $\frac{dy}{dx}$.

  And it is denoted by $m = \frac{dy}{dx} = \tan\theta$

- ✓ Slope of normal to the curve at a point (x,y) is $= \frac{-1}{\frac{dy}{dx}} = \frac{-1}{m} = m'$

Exercise 1.

Q.1. Find the slope of tangent and normal to the curve $y = x^2 - 6x + 3$ at point (6,3).

Q.2. Find the slope of tangent if $x = a\ cos^3\theta, y = b\ sin^3\theta$ at point $\theta = \frac{\pi}{4}$.

Q.3. Find the gradient of the tangent of the curve $y = \sqrt{x^3}\ at\ x = 4$.

Q.4. At which point on the curve $y = 3x - x^2$ the slope is -5.

Q.5. At which point on the curve $y = e^x$ the slope is 1.

Q.6. Find the point on the curve $y = \log(x - 3)$ at which slope is 5.

Q.7. Find the point on curve $y = x^3 - 3x + 3$, the tangent at which is parallel to x-axis.

Q.8. Find the point on curve $y = x^2 - 6x + 8$, the tangent at which is parallel to x-axis.

Q.9. Find the point on the curve $y = 2x^2 - 6x$, the tangent at which is parallel to x-axis.

Q.10. Find the point on the curve $y = 7x - 3x^2$ where tangent make an angle of 45°.

Q.11. Determine a & b such that slope of curve $2y^3 = ax^2 + b$ at $(1, -1)$is same as the slop of $x + y = 0$.

Q.12. The equation of tangent at the point $(2,3)$ on the curve $y = ax^3 + b$ is $y = 4x - 5$. Find the value of a and b.

Equation of Tangent and Normal:

✓ Equation of tangent at (x_1, y_1) on the curve in slope point form

$$(y - y_1) = m(x - x_1)$$

$$m = slope\ of\ tangent = \left(\frac{dy}{dx}\right)_{(x_1,y_1)}$$

✓ Equation of normal at (x_1, y_1) on the curve in slope point form

$$(y - y_1) = m'(x - x_1)$$

$$m' = slope\ of\ normal = \frac{-1}{m} = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}$$

Exercise 2.

Q.1. Find the equation of tangent and normal to the curve $y = x(2 - x)$ at the point $(2,0)$.

Q.2. Find the equation of tangent and normal to the curve $y = x^3 - 2x^2 + 4$ at the point $(2,4)$. OR Find the equation of tangent & normal to the curve $y = x^3 - 2x^2 + 4$ at the point $x = 2$.

Q.3. Find the equation of tangent and normal to the curve $4x^2 + 9y^2 = 40$ at the point $(1,2)$.

Q.4. Find the equation of normal to the curve $y = x^2 - x - 6$ at the point where it crosses x-axis.

Q.5. Find the equation of tangent and normal to the curve $y = 4xe^x$ at origin

Q.6. Find the equation of tangent and normal to the curve $13x^3 + 2x^2y + y^3 = 1$ at the point $(1, -2)$.

Q.7. Find the equation of tangent and normal to the curve $2x^2 - xy + 3y^2 = 18$ at the point $(3,1)$.

Q.8. Find the equation of tangent and normal to the ellipse $2x^2 + 3y^2 = 5$ which is perpendicular to the line $3x + 2y + 7 = 0$

Q.9. Find equation of tangent to the circle $x^2 + y^2 + 6x - 6y - 7 = 0$ at a point it cuts the axis.

Q.10. Find the equation of normal to the curve $3ay^2 = x^2(x + a)$ at the point $(2a, 2a)$.

Q.11. Find the equation of tangent $y^2(2a - x) = x^3$ at (a, a).

Q.12. Find the equation of tangent to the curve $x = \frac{1}{t}, y = t - \frac{1}{t}$ when $t = 2$.

Q.13. Find the equation of tangent to the curve $x = \frac{1}{t}, y = 1 - \frac{1}{t}$ when $t = 2$.

Q.14. Show that equation of tangent to $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 2$ at the point (a,b) is $\frac{x}{a} + \frac{y}{b} = 2$.

Maxima and Minima:

Exercise 3.

Q.1. Find the maximum and minimum of $x^3 - 9x^2 + 24x$.

Q.2. Find the maximum and minimum of $x^3 - 9x^2 + 24x - 7$.

Q.3. Find the maximum and minimum of $2x^3 - 3x^2 - 12x + 12$.

Q.4. Find the maximum and minimum of $2x^3 - 3x^2 - 36x + 10$.

Q.5. Find the value of x for which function is maximum and mini if $y = x^3 - \frac{15}{2}x^2 + 18x$

Q.6. Find the maximum and minimum value of $\tan x - 2x$.

Q.7. Find the maximum and minimum value of $y = \frac{4}{x+2} + x$

Q.8. Find the maximum and minimum value of $\left(\frac{4}{x} + \frac{36}{2-x}\right)$

Q.9. A metal wire 36cm long is bent to form a rectangle find its dimensions when its area is maximum.

Q.10. A metal wire 40cm long is bent to form a rectangle find its dimensions when its area is maximum.

Q.11. The perimeter of rectangle is 100M find the length of its side when area of rectangle is maximum.

Q.12. Divide 100 into two parts such that their product is maximum.

Q.13. Divide 80 into two parts such that their product is maximum.

Q.14. Divide 30 into two parts such that product of one and the cube of the other is maximum.

Q.15. A manufacture can sell x items at price of Rs.$(330 - x)$ each. The cost of producing x items in Rs. Is $x^2 + 10x + 12$. How many items must be sold so that his profit is maximum?

Radius of curvature (ρ)

Exercise 4.

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

Q.1. Find the radius of curvature of $y = x^3$ at point $(1,1)$

Q.2. Find the radius of curvature of $y = x^3 at\ (2,1)$

Q.3. Find the radius of curvature of $y = x^3 at\ (2,8)$

Q.4. Find the radius of curvature of $y = e^x at\ (0,1)$

Q.5. Find the radius of curvature of the curve $xy = c\ at\ (c,c)$

Q.6. Find the radius of curvature of $y = x^3 + 3x^2 + 2$ at (1,2)

Q.7. Find the radius of curvature of $y^2 = 4ax$ at origin.

Q.8. Find the radius of curvature of the curve $y^2 = 4ax\ at\ point\ (2,2a)$.

Q.9. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = 1\ at\ point\ \left(\frac{1}{4}, \frac{1}{4}\right)$.

Q.10. A beam is bent in the form of curve $y = 2\sin x - \sin 2x$. Find radius of curvature of the beam at $x = \frac{\pi}{2}$

Q.11. A telegram wire hang in the form of curve $y = a \log\left[sec\frac{x}{a}\right]$ where "a" is constant. Show that the radius of curvature at any point is $a \sec\left(\frac{x}{a}\right)$.

Q.12. Find the radius of curvature of the curve $y = \log(\sin x)$ $at\ x = \frac{\pi}{2}$

Q.13. Find the radius of curvature for the curve $x = a\ cos^3\theta,\ y = a\ sin^3\theta\ at\ \theta = \frac{\pi}{4}$.